Finite Mathematics

For the Managerial, Life, and Social Sciences

Eleventh Edition

TAN

A Sampling of New Applications

Drawn from diverse fields of interest and situations that occur in the real world

BUSINESS

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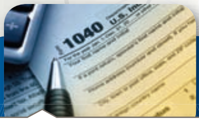

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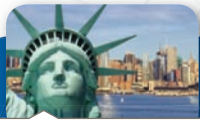

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FINITE MATHEMATICS

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

EDITION 11

FINITE MATHEMATICS

FOR THE MANAGERIAL, LIFE, AND SOCIAL SCIENCES

SOO T. TAN

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***Finite Mathematics: For the Managerial, Life, and Social Sciences*, Eleventh Edition**
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Compositor: Graphic World, Inc.

Library of Congress Control Number: 2013948993

ISBN-13: 978-1-285-46465-7

ISBN-10: 1-285-46465-6

Cengage Learning
200 First Stamford Place, 4th Floor
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Printed in the United States of America
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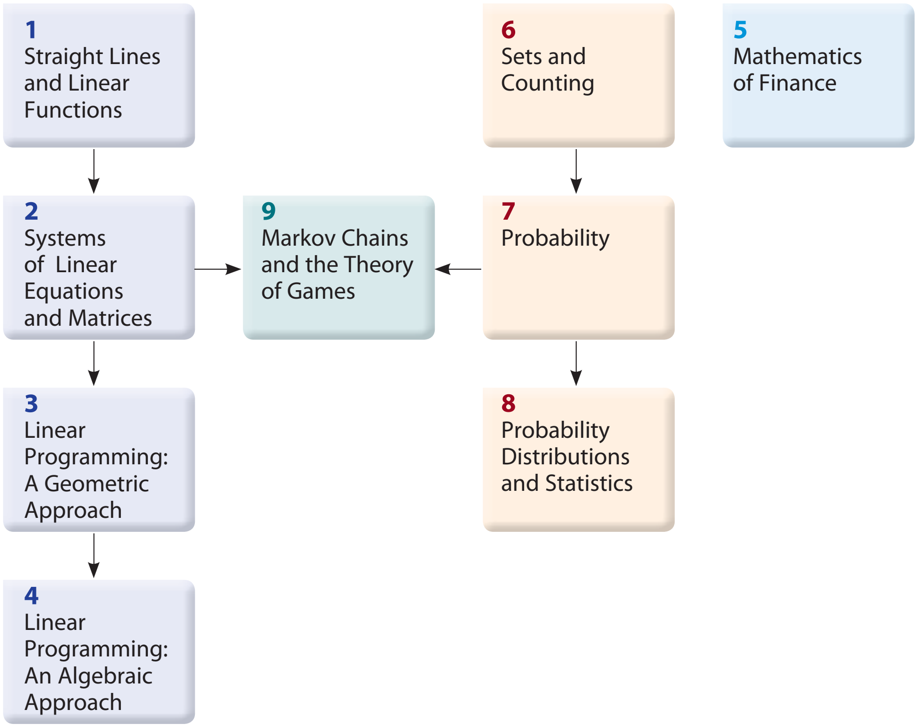
TO PAT, BILL, AND MICHAEL

CONTENTS

PREFACE

Math plays a vital role in our increasingly complex daily life. *Finite Mathematics for the Managerial, Life, and Social Sciences* attempts to illustrate this point with its applied approach to mathematics. Students have a much greater appreciation of the material if the applications are drawn from their fields of interest and from situations that occur in the real world. This is one reason you will see so many exercises in my texts that are modeled on data gathered from newspapers, magazines, journals, and other media. In addition, many students come into this course with some degree of apprehension. For this reason, I have adopted an intuitive approach in which I try to introduce each abstract mathematical concept through an example drawn from a common life experience. Once the idea has been conveyed, I then proceed to make it precise, thereby ensuring that no mathematical rigor is lost in this intuitive treatment of the subject.

The only prerequisite for understanding this text is one to two years, or the equivalent, of high school algebra. This text offers more than enough material for a one-semester or two-quarter course. The following chapter dependency chart is provided to help the instructor design a course that is most suitable for the intended audience.



The Approach

Presentation

Consistent with my intuitive approach, I state the results informally. However, I have taken special care to ensure that mathematical precision and accuracy are not compromised.

Motivation

Illustrating the practical value of mathematics in applied areas is an objective of my approach. Concepts are introduced with concrete, real-life examples wherever appropriate. These examples and other applications have been chosen from current topics and issues in the media and serve to answer a question often posed by students: "What will I ever use this for?"

Problem-Solving Emphasis

Special emphasis is placed on helping students formulate, solve, and interpret the results of applied problems. Because students often have difficulty setting up and solving word problems, extra care has been taken to help them master these skills:

- Very early on in the text, students are given practice in solving word problems.
- Guidelines are given to help students formulate and solve word problems.
- One entire section is devoted to modeling and setting up linear programming problems.

Modeling

One important skill that every student should acquire is the ability to translate a real-life problem into a mathematical model. In Section 1.3, the modeling process is discussed, and students are asked to use models (functions) constructed from real-life data to answer questions. Additionally, students get hands-on experience constructing these models in the Using Technology sections.

New to this Edition

The focus of this revision has been the continued emphasis on illustrating the mathematical concepts in *Finite Mathematics* by using more real-life applications that are relevant to the everyday life of students and to their fields of study in the managerial, life, and social sciences. A sampling of these new applications is provided on the inside front cover pages.

Many of the exercise sets have been revamped. In particular, the exercise sets were restructured to follow more closely the order of the presentation of the material in each section and to progress more evenly from easier to more difficult problems in both the rote and applied sections of each exercise set. Additional concept questions, rote exercises, and true-or-false questions were also included.

More Specific Content Changes

Chapter 1 In Section 1.2, parts (b) and (c) of Example 12 illustrate how to determine whether a point lies on a line. A new application, *Smokers in the United States*, has been added to the self-check exercises. In Section 1.3, new data have been used for the *U.S. Health-Care Expenditures* application, and students are shown how the new model for this application is constructed in Section 1.5 using the least-squares method. Also, in Using Technology Section 1.3, Applied Examples 2 and 4, *Drinking and Driving Among High School Students* have been added for the graphing calculator and Excel applications.

Chapters 2–4 A wealth of new application exercises has been added, and many examples and exercises have been updated. Also, in Section 3.1, newly added Example 6 illustrates how to determine whether a point lies in a feasible set of inequalities. This is followed by a new application, Applied Example 7, *A Production Problem,* in which students are shown how they can use a solution set for a given system of inequalities (restrictions) to determine whether certain production goals can be met. Also, in Section 3.1, Exercise 44, we see how the solution of a system of linear equations is obtained by looking at a system of inequalities.

Chapter 5 Interest rate problems throughout the entire chapter were revised to reflect the current interest rate environment. Also, in Section 5.3, two new exercises were added illustrating the new *Ability-to-Repay Rules for Mortgages* adopted by the Consumer Financial Protection Bureau in response to the recent financial crisis.

Chapters 6–8 In the probability and statistics chapters of the text, the emphasis is again placed on providing new real-life application exercises. These chapters deal with the calculations of probabilities and data analysis and the emphasis here was placed on providing data from marketing, economic, consumer, and scientific surveys that was relevant, current, and of interest to students to motivate the mathematical concepts presented. Some of these surveys involve the following questions:

What is the greatest challenge upon starting a new job?

How many years will it take you to fully recover from the Great Recession?

What is the most common cause of on-the-job distraction?

How do workers get ahead on the job?

How many social media accounts do you have?

Have gas prices caused you any financial hardship?

Also, several new examples were added in these chapters: In Section 6.1, Applied Example 13, *Cyber Privacy,* illustrates set operations. In Section 6.2, Example 5 illustrates how the solution of a system of linear equations can sometimes be used to help draw a Venn diagram. In Section 7.5, Example 8 illustrates the difference between mutually exclusive and independent events. Also, Applied Example 12, *Predicting Travel Weather,* illustrates the calculation of the probability of independent events. In Section 8.2, Applied Example 8, *Commuting Times,* illustrates the calculation of expected value for grouped data; and in Section 8.3, Applied Example 4, *Married Men,* illustrates the calculation of standard deviation for grouped data. Using Technology Section 8.1 was expanded to include an example (Applied Example 3, *Time Use of College Students*) and exercises illustrating how Excel can be used to create pie charts.

Features

Real-World Connections

Motivating Applications
Many new applied examples and exercises have been added in the Eleventh Edition. Among the topics of the new applications are Facebook users, satellite TV subscribers, criminal justice, cyber privacy, brand switching among college students, social media accounts, detecting shoplifters, and smartphone ownership.

APPLIED EXAMPLE 13 Cyber Privacy In a poll surveying 1500 registered voters in California, the respondents were asked to rank the following companies on a scale of 0 to 10 in terms of how much they could trust these companies to keep their personal information secure, with zero meaning that they don't trust the company.

Company	Apple	Google	LinkedIn	YouTube	Facebook	Twitter
Rating	4.6	3.8	3.0	2.8	2.7	2.4

Let A denote the set of companies that have a rating higher than 2.5, let B denote the set of companies that have a rating between 2.5 and 4, and let C denote the set of companies that have a rating lower than 3. Find the following sets:

a. A, B, and C **b.** $A \cup B$ **c.** $B \cap C$ **d.** $A^c \cap B$ **e.** $A \cap B^c$

Source: Los Angeles Times.

Portfolios
These interviews share the varied experiences of professionals who use mathematics in the workplace. Among those included are a city manager at a photography company and a technical director at a wireless company who uses his knowledge of game theory to help mobile operators develop and deliver new technologies.

PORTFOLIO Christian Derrick

TITLE Technical Director, Europe
INSTITUTION SpiderCloud Wireless

SpiderCloud Wireless is the start-up company that introduced the first enterprise radio access network (E-RAN) systems into mobile networks. E-RAN technology allows for superior indoor coverage and capacity as well as high-value services for a mobile operator's most important customer, the business user. As the Technical Director, Europe, I am responsible for working with European mobile operators to develop and deliver new wireless technologies.

At SpiderCloud, we draw on our knowledge of mathematics to understand the strategic decisions of our mobile partners. Applying our knowledge of game theory, for example, we can begin to answer a variety of questions such as: How will mobile operators price their voice, data, and SMS plans? And how will suppliers price their cell phone equipment?

Let's consider the first question. If each mobile operator offered identical pricing plans, one would predict that a price reduction by an operator would lead to an increase in that operator's market share. But price, of course, is not the only factor a consumer takes into consideration when selecting a plan. Consumers also consider other variables such as the operator's brand name and reputation, the device that is bundled with the plan, the geographical coverage of the operator's network, and a range of add-on options.

Game theory helps mobile operators to factor the perceived value of these factors into their pricing plans. Operators use game theory to evaluate the likely outcomes of changes in pricing in response to both their competitors' actions and their competitors' responses to their own actions. This helps operators to maximize their profits. Working at SpiderCloud, I have learned that an in-depth knowledge of game theory, including Nash equilibrium and the prisoner's dilemma, is extremely useful for predicting the behavior of players in a complex business environment.

Michelle Preen; (inset) © dani3315/ShutterStock.com

Explorations and Technology

Explore and Discuss
These optional questions can be discussed in class or assigned as homework. They generally require more thought and effort than the usual exercises. They may also be used to add a writing component to the class or as team projects.

Explore and Discuss

Let A, B, and C be nonempty subsets of a set U.

1. Suppose $A \cap B \neq \varnothing$, $A \cap C \neq \varnothing$, and $B \cap C \neq \varnothing$. Can you conclude that $A \cap B \cap C \neq \varnothing$? Explain your answer with an example.
2. Suppose $A \cap B \cap C \neq \varnothing$. Can you conclude that $A \cap B \neq \varnothing$, $A \cap C \neq \varnothing$, and $B \cap C \neq \varnothing$? Explain your answer.

Exploring with Technology
These optional discussions appear throughout the main body of the text and serve to enhance the student's understanding of the concepts and theory presented. Often the solution of an example in the text is augmented with a graphical or numerical solution.

Exploring with TECHNOLOGY

To obtain a visual confirmation of the fact that the expression $\left(1 + \frac{1}{u}\right)^u$ approaches the number $e = 2.71828\ldots$ as u gets larger and larger, plot the graph of $f(x) = \left(1 + \frac{1}{x}\right)^x$ in a suitable viewing window, and observe that $f(x)$ approaches $2.71828\ldots$ as x gets larger and larger. Use ZOOM and TRACE to find the value of $f(x)$ for large values of x.

Using Technology
Written in the traditional example-exercise format, these optional sections show how to use the graphing calculator and Microsoft Excel 2010 as a tool to solve problems. (Instructions for Microsoft Excel 2007 are given on the companion website.) Illustrations showing graphing calculator screens and spreadsheets are used extensively. In keeping with the theme of motivation through real-life examples, many sourced applications are included.

A *How-To Technology Index* is included at the back of the book for easy reference to Using Technology examples.

APPLIED EXAMPLE 3 Time Use of College Students Use the data given in Table 1 to construct a pie chart.

TABLE T1

Time Used on an Average Weekday for Full-Time University and College Students

Time Use	Time (in hours)
Sleeping	8.5
Leisure and sports	3.7
Working and related activities	2.9
Educational activities	3.3
Eating and drinking	1.0
Grooming	0.7
Traveling	1.5
Other	2.4

Source: Bureau of Labor Statistics.

Solution

We begin by entering the information from Table T1 in Columns A and B on a spreadsheet. Then follow these steps:

Step 1 First, highlight the data in cells A2:A9 and B2:B9 as shown in Figure T4.

	A	B
1	Time Use	Time (in hours)
2	Sleeping	8.5
3	Leisure	
4	Workin	
5	Educati	
6	Eating a	
7	Groomi	
8	Travelir	
9	Other	

FIGURE T4 Completed sp

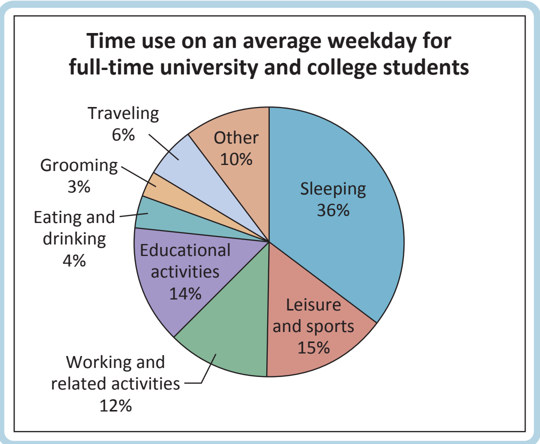


FIGURE T5
The pie chart describing the data in Table T1

Concept Building and Critical Thinking

Self-Check Exercises

Offering students immediate feedback on key concepts, these exercises begin each end-of-section exercise set and contain both rote and word problems (applications). Fully worked-out solutions can be found at the end of each exercise section. If students get stuck while solving these problems, they can get immediate help before attempting to solve the homework exercises. Applications have been included here because students often need extra practice with setting up and solving these problems.

1.2 Self-Check Exercises

1. Determine the number a such that the line passing through the points $(a, 2)$ and $(3, 6)$ is parallel to a line with slope 4.
2. Find an equation of the line that passes through the point $(3, -1)$ and is perpendicular to a line with slope $-\frac{1}{2}$.
3. Does the point $(3, -3)$ lie on the line with equation $2x - 3y - 12 = 0$? Sketch the graph of the line.
4. **Smokers in the United States** The following table gives the percentage of adults in the United States from 2006 through 2010 who smoked in year t. Here, $t = 0$ corresponds to the beginning of 2006.

Year, t	0	1	2	3	4
Percent, y	20.8	20.5	20.1	19.8	19.0

   a. Plot the percentage of U.S. adults who smoke (y) versus the year (t) for the given years.
   b. Draw the line L through the points $(0, 20.8)$ and $(4, 19.0)$.
   c. Find an equation of the line L.
   d. Assuming that this trend continues, estimate the percentage of U.S. adults who smoked at the beginning of 2014.

Source: Centers for Disease Control and Prevention.

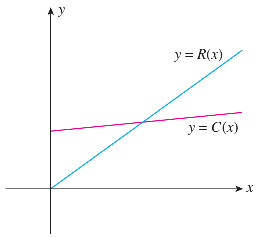
Solutions to Self-Check Exercises 1.2 can be found on page 24.

Concept Questions

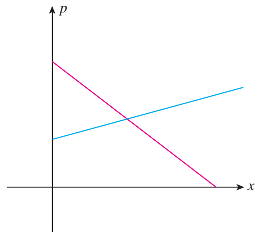
Designed to test students' understanding of the basic concepts discussed in the section, these questions encourage students to explain learned concepts in their own words.

1.4 Concept Questions

1. Explain why you would expect that the intersection of a linear demand curve and a linear supply curve would lie in the first quadrant.
2. In the accompanying figure, $C(x)$ is the cost function and $R(x)$ is the revenue function associated with a certain product.
   a. Plot the break-even point $P(x_0, y_0)$ on the graph.
   b. Identify and mark the break-even quantity, x_0, and the break-even revenue, y_0, on the set of axes.



3. The accompanying figure gives the demand curve and the supply curve associated with a certain commodity.



   a. Identify the demand curve and the supply curve.
   b. Plot the point $P(x_0, p_0)$ that corresponds to market equilibrium.
   c. Identify and mark the equilibrium quantity, x_0, and the equilibrium price, p_0, on the set of axes.

Exercises

Each section contains an ample set of exercises of a routine computational nature followed by an extensive set of modern application exercises.

1.4 Exercises

In Exercises 1–6, find the point of intersection of each pair of straight lines.

1. $y = 3x + 4$; $y = -2x + 14$
2. $y = -4x - 7$; $-y = 5x + 10$
3. $2x - 3y = 6$; $3x + 6y = 16$
4. $2x + 4y = 11$; $-5x + 3y = 5$
5. $y = \frac{1}{4}x - 5$; $2x - \frac{3}{2}y = 1$
6. $y = \frac{2}{3}x - 4$; $x + 3y + 3 = 0$

In Exercises 7–10, find the break-even point for the firm whose cost function C and revenue function R are given.

7. $C(x) = 5x + 10{,}000$; $R(x) = 15x$
8. $C(x) = 15x + 12{,}000$; $R(x) = 21x$

   a. Find the functions describing the daily cost of leasing from each company.
   b. Sketch the graphs of the two functions on the same set of axes.
   c. If a customer plans to drive at most 30 mi, from which company should he rent a truck for a single day?
   d. If a customer plans to drive at least 60 mi, from which company should he rent a truck for a single day?

15. **Decision Analysis** A product may be made by using Machine I or Machine II. The manufacturer estimates that the monthly fixed costs of using Machine I are \$18,000, whereas the monthly fixed costs of using Machine II are \$15,000. The variable costs of manufacturing 1 unit of the product using Machine I and Machine II are \$15 and \$20, respectively. The product sells for \$50 each.
   a. Find the cost functions associated with using each machine.

Review and Study Tools

Summary of Principal Formulas and Terms

Each review section begins with the Summary, which highlights the important equations and terms, with page numbers given for quick review.

CHAPTER 6 Summary of Principal Formulas and Terms

FORMULAS

1. Commutative laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
2. Associative laws	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
3. Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. De Morgan's laws	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$
5. Number of elements in the union of two finite sets	$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
6. Permutation of n distinct objects, taken r at a time	$P(n, r) = \dfrac{n!}{(n - r)!}$
7. Permutation of n objects, not all distinct, taken n at a time	$\dfrac{n!}{n_1!\, n_2! \cdots n_m!}$
8. Combination of n distinct objects, taken r at a time	$C(n, r) = \dfrac{n!}{r!\,(n - r)!}$

TERMS

set (342)
element of a set (342)
roster notation (342)
set-builder notation (342)
set equality (342)
subset (343)
empty set (343)
universal set (344)
Venn diagram (344)
set union (345)
set intersection (345)
complement of a set (345)
set complementation (346)
multiplication principle (362)
generalized multiplication principle (363)
permutation (368)
n-factorial (370)
combination (374)

Concept Review Questions

These questions give students a chance to check their knowledge of the basic definitions and concepts given in each chapter.

CHAPTER 6 Concept Review Questions

Fill in the blanks.

1. A well-defined collection of objects is called a/an _____. These objects are called _____ of the _____.
2. Two sets having exactly the same elements are said to be _____.
3. If every element of a set A is also an element of a set B, then A is a/an _____ of B.
4. **a.** The empty set $\varnothing$ is the set containing _____ elements.
   b. The universal set is the set containing _____ elements.

Review Exercises

Offering a solid review of the chapter material, the Review Exercises contain routine computational exercises followed by applied problems.

CHAPTER 6 Review Exercises

In Exercises 1–4, list the elements of each set in roster notation.

1. $\{x \mid 3x - 2 = 7 \text{ and } x \text{ is an integer}\}$
2. $\{x \mid x$ is a letter of the word *TALLAHASSEE*$\}$
3. The set whose elements are the even numbers between 3 and 11
4. $\{x \mid (x - 3)(x + 4) = 0 \text{ and } x \text{ is a negative integer}\}$

Let $A = \{a, c, e, r\}$. In Exercises 5–8, determine whether the set is equal to A.

5. $\{r, e, c, a\}$
6. $\{x \mid x$ is a letter of the word *career*$\}$
7. $\{x \mid x$ is a letter of the word *racer*$\}$
8. $\{x \mid x$ is a letter of the word *cares*$\}$

For Exercises 17–20, let

U = {all participants in a consumer-behavior survey conducted by a national polling group}
A = {consumers who avoided buying a product because it is not recyclable}
B = {consumers who used cloth rather than disposable diapers}
C = {consumers who boycotted a company's products because of its record on the environment}
D = {consumers who voluntarily recycled their garbage}

Describe each set in words.

17. $A \cap C$ **18.** $A \cup D$
19. $B^c \cap D$ **20.** $C^c \cup D^c$

Before Moving On . . .

Found at the end of each chapter review, these exercises give students a chance to determine whether they have mastered the basic computational skills developed in the chapter.

CHAPTER 6 Before Moving On . . .

1. Let $U = \{a, b, c, d, e, f, g\}$, $A = \{a, d, f, g\}$, $B = \{d, f, g\}$, and $C = \{b, c, e, f\}$. Find:
   a. $A \cap (B \cup C)$
   b. $(A \cap C) \cup (B \cup C)$
   c. A^c
2. Let A, B, and C be subsets of a universal set U, and suppose that $n(U) = 120$, $n(A) = 20$, $n(A \cap B) = 10$, $n(A \cap C) = 11$, $n(B \cap C) = 9$, and $n(A \cap B \cap C) = 4$. Find $n[A \cap (B \cup C)^c]$.
3. In how many ways can four compact discs be selected from six different compact discs?
4. From a standard 52-card deck, how many 5-card poker hands can be dealt consisting of 3 deuces and 2 face cards?
5. There are six seniors and five juniors in the Chess Club at Madison High School. In how many ways can a team consisting of three seniors and two juniors be selected from the members of the Chess Club?

Action-Oriented Study Tabs

Convenient color-coded study tabs make it easy for students to flag pages that they want to return to later, whether for additional review, exam preparation, online exploration, or identifying a topic to be discussed with the instructor.

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Acknowledgments

I wish to express my personal appreciation to each of the following reviewers, whose many suggestions have helped make a much improved book.

Zach Abernathy
Winthrop University

Mark Antkowicz
Colorado Technical University

Andrea Brown
Ivy Tech Community College

Ashot Djrbashian
Glendale Community College

Amy Franklin
Jacksonville State University

Jean Johnson
Jacksonville State University

Peter Knopf
Pace University

Thurai Kugan
John Jay College of Criminal Justice, CUNY

Melanie Ledwig
Victoria College

Myra Maxwell
University of Indianapolis

Laurie McManus
St. Louis Community College at Meramec

Markus Pomper
Indiana University East

John Roepke
Doane College

Peter Shenkin
John Jay College of Criminal Justice, CUNY

Beimnet Teclezghi
New Jersey City University

Brenda F. Tiefenbruck
University of St. Thomas

Kenneth V. Turner, Jr.
Anderson University

Francis J. Vasko
Kutztown University

Mary Wright
Southern Illinois University Carbondale

Jill Zarestky
Texas A&M University

I also thank reviewers of previous editions whose comments and suggestions have helped to get the book this far.

I also wish to thank Tao Guo for the excellent job he did as the accuracy checker for this text. I also thank the editorial and production staffs of Cengage Learning—Richard Stratton, Rita Lombard, Laura Wheel, Jennifer Cordoba, Andrew Coppola, Cheryll Linthicum, and Vernon Boes—for all of their help and support during the development and production of this edition. I also thank Martha Emry and Barbara Willette, who both did an excellent job ensuring the accuracy and readability of this edition. Simply stated, the team I have been working with is outstanding, and I truly appreciate all of their hard work and efforts.

S. T. Tan

About the Author

SOO T. TAN received his S.B. degree from Massachusetts Institute of Technology, his M.S. degree from the University of Wisconsin–Madison, and his Ph.D. from the University of California at Los Angeles. He has published numerous papers in optimal control theory, numerical analysis, and mathematics of finance. He is also the author of a series of calculus textbooks.

1 Straight Lines and Linear Functions

THIS CHAPTER INTRODUCES the Cartesian coordinate system, a system that allows us to represent points in the plane in terms of ordered pairs of real numbers. This in turn enables us to compute the distance between two points algebraically. We also study straight lines. *Linear functions,* whose graphs are straight lines, can be used to describe many relationships between two quantities. These relationships can be found in fields of study as diverse as business, economics, the social sciences, physics, and medicine. In addition, we see how some practical problems can be solved by finding the point(s) of intersection of two straight lines. Finally, we learn how to find an algebraic representation of the straight line that "best" fits a set of data points that are scattered about a straight line.

Because the over-65 population will be growing more rapidly in the next few decades, U.S. health-care expenditures are expected to be boosted significantly. What will be the rate of increase of these expenditures over the next few years? How much will health-care expenditures be in 2014? In Example 1, page 31, we use a mathematical model based on figures from the Centers for Medicare & Medicaid Services to answer these questions.


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1.1 The Cartesian Coordinate System

The Cartesian Coordinate System

The real number system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We assume that you are familiar with the rules governing these algebraic operations (see Appendix B).

Real numbers may be represented geometrically by points on a line. This line is called the **real number,** or **coordinate, line.** We can construct the real number line as follows: Arbitrarily select a point on a straight line to represent the number 0. This point is called the **origin.** If the line is horizontal, then choose a point at a convenient distance to the right of the origin to represent the number 1. This determines the scale for the number line. Each positive real number x lies x units to the right of 0, and each negative real number x lies $-x$ units to the left of 0.

In this manner, a one-to-one correspondence is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (Figure 1).

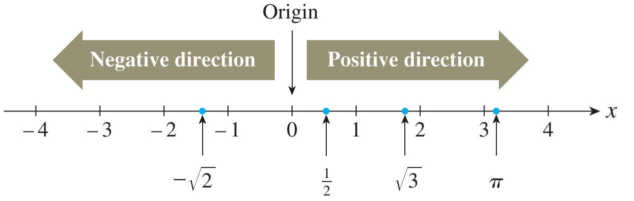


FIGURE 1
The real number line

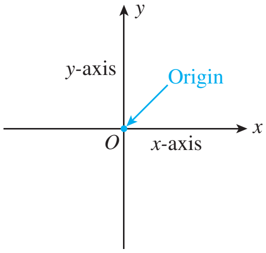


FIGURE 2
The Cartesian coordinate system

In a similar manner, we can represent points in a plane (a two-dimensional space) by using the **Cartesian coordinate system,** which we construct as follows: Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O, called the **origin** (Figure 2). The horizontal line is called the ***x*-axis,** and the vertical line is called the ***y*-axis.** A number scale is set up along the x-axis, with the positive numbers lying to the right of the origin and the negative numbers lying to the left of it. Similarly, a number scale is set up along the y-axis, with the positive numbers lying above the origin and the negative numbers lying below it.

Note The number scales on the two axes need not be the same. Indeed, in many applications, different quantities are represented by x and y. For example, x may represent the number of smartphones sold, and y may represent the total revenue resulting from the sales. In such cases, it is often desirable to choose different number scales to represent the different quantities. Note, however, that the zeros of both number scales coincide at the origin of the two-dimensional coordinate system.

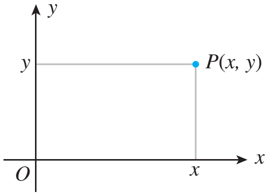


FIGURE 3
An ordered pair in the coordinate plane

We can represent a point in the plane in this coordinate system by an **ordered pair** of numbers—that is, a pair (x, y) in which x is the first number and y is the second. To see this, let P be any point in the plane (Figure 3). Draw perpendicular lines from P to the x-axis and y-axis, respectively. Then the number x is precisely the number that corresponds to the point on the x-axis at which the perpendicular line through P hits the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the perpendicular line through P crosses the y-axis.

Conversely, given an ordered pair (x, y) with x as the first number and y as the second, a point P in the plane is uniquely determined as follows: Locate the point on the x-axis represented by the number x, and draw a line through that point perpendicular to the x-axis. Next, locate the point on the y-axis represented by the number y, and draw a line through that point perpendicular to the y-axis. The point of intersection of these two lines is the point P (Figure 3).

In the ordered pair (x, y), x is called the **abscissa,** or ***x*-coordinate;** y is called the **ordinate,** or ***y*-coordinate;** and x and y together are referred to as the **coordinates** of the point P. The point P with x-coordinate equal to a and y-coordinate equal to b is often written $P(a, b)$.

The points $A(2, 3)$, $B(-2, 3)$, $C(-2, -3)$, $D(2, -3)$, $E(3, 2)$, $F(4, 0)$, and $G(0, -5)$ are plotted in Figure 4.

Note In general, $(x, y) \neq (y, x)$. This is illustrated by the points A and E in Figure 4.

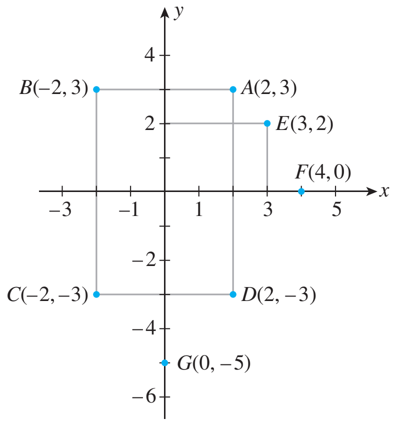


FIGURE 4
Several points in the coordinate plane

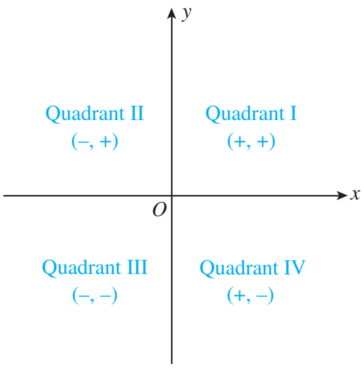


FIGURE 5
The four quadrants in the coordinate plane

The axes divide the plane into four quadrants. Quadrant I consists of the points P with coordinates x and y, denoted by $P(x, y)$, satisfying $x > 0$ and $y > 0$; Quadrant II consists of the points $P(x, y)$ where $x < 0$ and $y > 0$; Quadrant III consists of the points $P(x, y)$ where $x < 0$ and $y < 0$; and Quadrant IV consists of the points $P(x, y)$ where $x > 0$ and $y < 0$ (Figure 5).

The Distance Formula

One immediate benefit that arises from using the Cartesian coordinate system is that the distance between any two points in the plane may be expressed solely in terms of the coordinates of the points. Suppose, for example, (x_1, y_1) and (x_2, y_2) are any two points in the plane (Figure 6). Then we have the following:

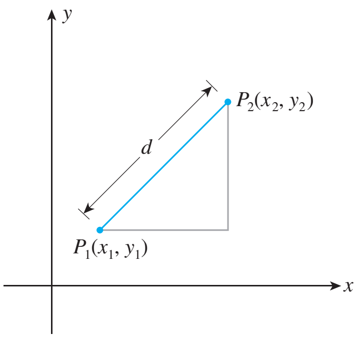


FIGURE 6
The distance between two points in the coordinate plane

Distance Formula

The distance d between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{1}$$

For a proof of this result, see Exercise 49, page 9.

Explore and Discuss

Refer to Example 1. Suppose we label the point $(2, 6)$ as P_1 and the point $(-4, 3)$ as P_2. (1) Show that the distance d between the two points is the same as that obtained in Example 1. (2) Prove that, in general, the distance d in Formula (1) is independent of the way we label the two points.

In what follows, we give several applications of the distance formula.

EXAMPLE 1 Find the distance between the points $(-4, 3)$ and $(2, 6)$.

Solution Let $P_1(-4, 3)$ and $P_2(2, 6)$ be points in the plane. Then we have

$$x_1 = -4 \quad \text{and} \quad y_1 = 3$$
$$x_2 = 2 \qquad\qquad y_2 = 6$$

Using Formula (1), we have

$$\begin{aligned} d &= \sqrt{[2-(-4)]^2 + (6-3)^2} \\ &= \sqrt{6^2 + 3^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

APPLIED EXAMPLE 2 **The Cost of Laying Cable** In Figure 7, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on a nearby island. A cable is to be laid connecting the relay station at S with the experimental station at M via the point Q that lies on the x-axis between O and S. If the cost of running the cable on land is \$3 per running foot and the cost of running the cable underwater is \$5 per running foot, find the total cost for laying the cable.

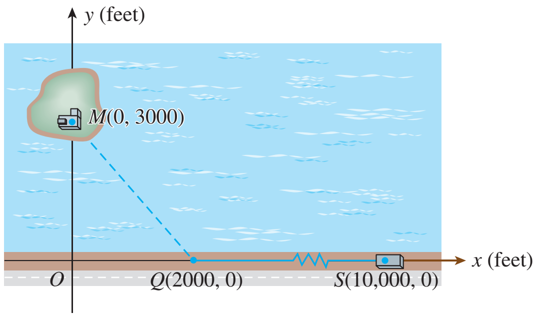


FIGURE 7
The cable will connect the relay station S to the experimental station M.

Solution The length of cable required on land is given by the distance from S to Q. This distance is $(10{,}000 - 2000)$, or 8000 feet. Next, we see that the length of cable required underwater is given by the distance from Q to M. This distance is

$$\begin{aligned} \sqrt{(0-2000)^2 + (3000-0)^2} &= \sqrt{2000^2 + 3000^2} \\ &= \sqrt{13{,}000{,}000} \\ &\approx 3606 \end{aligned}$$

or approximately 3606 feet. Therefore, the total cost for laying the cable is approximately

$$3(8000) + 5(3606) \approx 42{,}030$$

dollars.

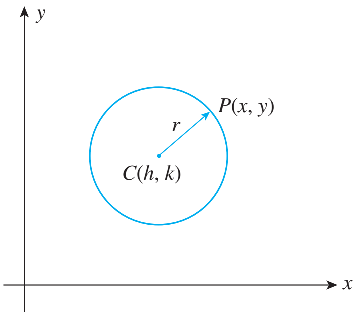


FIGURE **8**
A circle with radius r and center $C(h, k)$

EXAMPLE 3 Let $P(x, y)$ denote a point lying on a circle with radius r and center $C(h, k)$ (Figure 8). Find a relationship between x and y.

Solution By the definition of a circle, the distance between $C(h, k)$ and $P(x, y)$ is r. Using Formula (1), we have

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

which, upon squaring both sides, gives the equation

$$(x-h)^2 + (y-k)^2 = r^2$$

which must be satisfied by the variables x and y.

A summary of the result obtained in Example 3 follows.

Equation of a Circle

An equation of the circle with center $C(h, k)$ and radius r is given by

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \textbf{(2)}$$

EXAMPLE 4 Find an equation of the circle with (a) radius 2 and center $(-1, 3)$ and (b) radius 3 and center located at the origin.

Solution

a. We use Formula (2) with $r = 2$, $h = -1$, and $k = 3$, obtaining

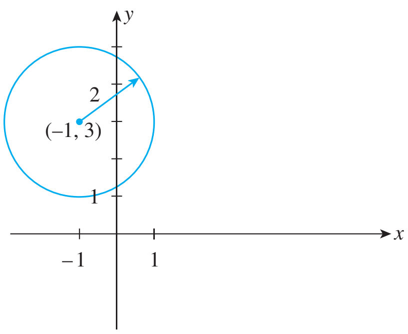
$$[x-(-1)]^2 + (y-3)^2 = 2^2$$
$$(x+1)^2 + (y-3)^2 = 4$$

(Figure 9a).

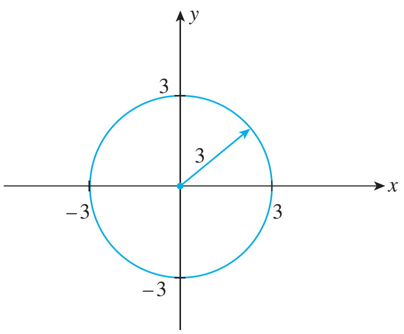
b. Using Formula (2) with $r = 3$ and $h = k = 0$, we obtain

$$x^2 + y^2 = 3^2$$
$$x^2 + y^2 = 9$$

(Figure 9b).



(a) The circle with radius 2 and center $(-1, 3)$



(b) The circle with radius 3 and center (0, 0)

FIGURE **9**

Explore and Discuss

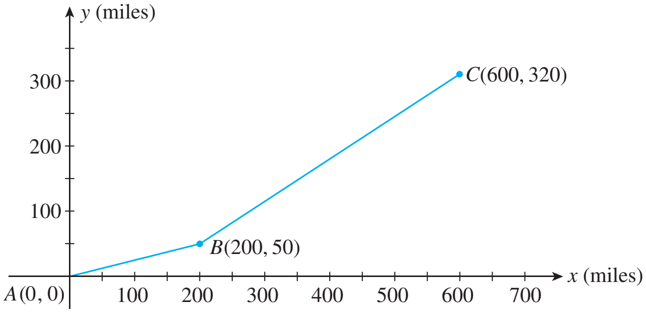
1. Use the distance formula to help you describe the set of points in the xy-plane satisfying each of the following inequalities, where $r > 0$.
   a. $(x - h)^2 + (y - k)^2 \le r^2$
   b. $(x - h)^2 + (y - k)^2 < r^2$
   c. $(x - h)^2 + (y - k)^2 \ge r^2$
   d. $(x - h)^2 + (y - k)^2 > r^2$
2. Consider the equation $x^2 + y^2 = 4$.
   a. Show that $y = \pm\sqrt{4 - x^2}$.
   b. Describe the set of points (x, y) in the xy-plane satisfying the equation

   (i) $y = \sqrt{4 - x^2}$ (ii) $y = -\sqrt{4 - x^2}$

1.1 Self-Check Exercises

1. **a.** Plot the points $A(4, -2)$, $B(2, 3)$, and $C(-3, 1)$.
   b. Find the distance between the points A and B, between B and C, and between A and C.
   c. Use the Pythagorean Theorem to show that the triangle with vertices A, B, and C is a right triangle.

2. **Fuel Stop Planning** The accompanying figure shows the location of Cities A, B, and C. Suppose a pilot wishes to fly from City A to City C but must make a mandatory stopover in City B. If the single-engine light plane has a range of 650 mi, can the pilot make the trip without refueling in City B?

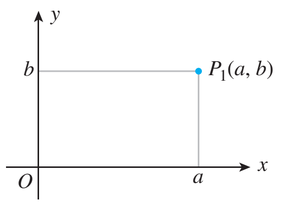


Solutions to Self-Check Exercises 1.1 can be found on page 10.

1.1 Concept Questions

1. What can you say about the signs of a and b if the point $P(a, b)$ lies in (a) the second quadrant? (b) The third quadrant? (c) The fourth quadrant?

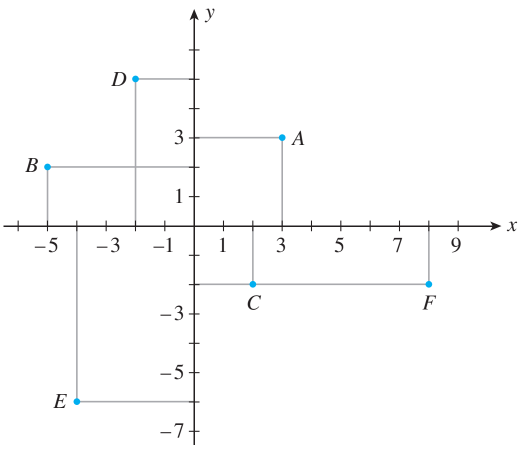
2. Refer to the accompanying figure.



   a. Given the point $P_1(a, b)$, where $a > 0$ and $b > 0$, plot the points $P_2(-a, b)$, $P_3(-a, -b)$, and $P_4(a, -b)$.
   b. What can you say about the distance of the points $P_1(a, b)$, $P_2(-a, b)$, $P_3(-a, -b)$, and $P_4(a, -b)$ from the origin?

1.1 Exercises

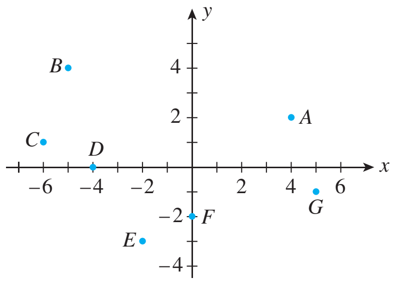
In Exercises 1–6, refer to the accompanying figure and determine the coordinates of the point and the quadrant in which it is located.



1. A **2.** B **3.** C

4. D **5.** E **6.** F

In Exercises 7–12, refer to the accompanying figure.



7. Which point is represented by the ordered pair $(4, 2)$?

8. What are the coordinates of point B?

9. Which points have negative y-coordinates?

10. Which point has a negative x-coordinate and a negative y-coordinate?

11. Which point has an x-coordinate that is equal to zero?

12. Which point has a y-coordinate that is equal to zero?

In Exercises 13–20, sketch a set of coordinate axes and then plot the point.

13. $(-2, 5)$ **14.** $(1, 3)$

15. $(3, -1)$ **16.** $(3, -4)$

17. $\left(8, -\frac{7}{2}\right)$ **18.** $\left(-\frac{5}{2}, \frac{3}{2}\right)$

19. $(4.5, -4.5)$ **20.** $(1.2, -3.4)$

In Exercises 21–24, find the distance between the points.

21. $(1, 3)$ and $(4, 7)$

22. $(1, 0)$ and $(4, 4)$

23. $(-1, 3)$ and $(4, 9)$

24. $(-2, 1)$ and $(10, 6)$

25. Find the coordinates of the points that are 10 units away from the origin and have a y-coordinate equal to -6.

26. Find the coordinates of the points that are 5 units away from the origin and have an x-coordinate equal to 3.

27. Show that the points $(3, 4)$, $(-3, 7)$, $(-6, 1)$, and $(0, -2)$ form the vertices of a square.

28. Show that the triangle with vertices $(-5, 2)$, $(-2, 5)$, and $(5, -2)$ is a right triangle.

In Exercises 29–34, find an equation of the circle that satisfies the given conditions.

29. Radius 5 and center $(2, -3)$

30. Radius 3 and center $(-2, -4)$

31. Radius 5 and center at the origin

32. Center at the origin and passes through $(2, 3)$

33. Center $(2, -3)$ and passes through $(5, 2)$

34. Center $(-a, a)$ and radius $2a$

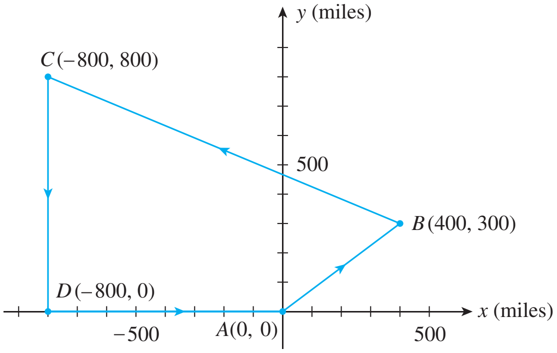
35. **TRACKING A CRIMINAL WITH GPS** After obtaining a warrant, the police attached a GPS tracking device to the car of a murder suspect. Suppose the car was located at the origin of a Cartesian coordinate system when the device was attached. Shortly afterwards, the suspect's car was tracked going 5 mi due east, 4 mi due north, and 1 mi due west before coming to a permanent stop.

a. What are the coordinates of the suspect's car at its final destination?

b. What was the distance traveled by the suspect?

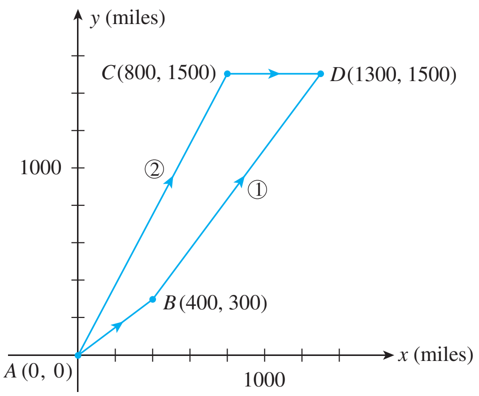
c. What is the distance as the crow flies between the original position and the final position of the suspect's car?

36. Planning a Grand Tour A grand tour of four cities begins at City A and makes successive stops at Cities B, C, and D before returning to City A. If the cities are located as shown in the accompanying figure, find the total distance covered on the tour.



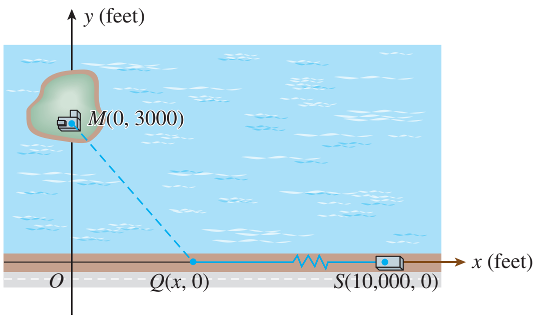
37. Will You Incur a Delivery Charge? A furniture store offers free setup and delivery services to all points within a 25-mi radius of its warehouse distribution center. If you live 20 mi east and 14 mi south of the warehouse, will you incur a delivery charge? Justify your answer.

38. Optimizing Travel Time Towns A, B, C, and D are located as shown in the accompanying figure. Two highways link Town A to Town D. Route 1 runs from Town A to Town D via Town B, and Route 2 runs from Town A to Town D via Town C. If a salesman wishes to drive from Town A to Town D and traffic conditions are such that he could expect to average the same speed on either route, which highway should he take to arrive in the shortest time?



39. Minimizing Shipping Costs for a Fleet of Autos Refer to the figure for Exercise 38. Suppose a fleet of 100 automobiles are to be shipped from an assembly plant in Town A to Town D. They may be shipped either by freight train along Route 1 at a cost of 66¢/mile/automobile or by truck along Route 2 at a cost of 62¢/mile/automobile. Which means of transportation minimizes the shipping cost? What is the net savings?

40. Cost of Laying Cable In the accompanying diagram, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on a nearby island. A cable is to be laid connecting the relay station at S with the experimental station at M via the point Q that lies on the x-axis between O and S. If the cost of running the cable on land is \$3/running foot and the cost of running cable underwater is \$5/running foot, find an expression in terms of x that gives the total cost of laying the cable. Use this expression to find the total cost when $x = 1500$ and when $x = 2500$.



41. Purchasing an HDTV Antenna Will Barclay wishes to determine which HDTV antenna he should purchase for his home. The TV store has supplied him with the following information:

Range in Miles			
VHF	**UHF**	**Model**	**Price**
30	20	A	\$50
45	35	B	60
60	40	C	70
75	55	D	80

Will wishes to receive Channel 17 (VHF), which is located 25 mi east and 35 mi north of his home, and Channel 38 (UHF), which is located 20 mi south and 32 mi west of his home. Which model will allow him to receive both channels at the least cost? (Assume that the terrain between Will's home and both broadcasting stations is flat.)

42. Distance Between Two Cruise Ships Two cruise ships leave port at the same time. Ship A sails north at a speed of 20 mph while Ship B sails east at a speed of 30 mph.

a. Find an expression in terms of the time t (in hours) giving the distance between the two cruise ships.

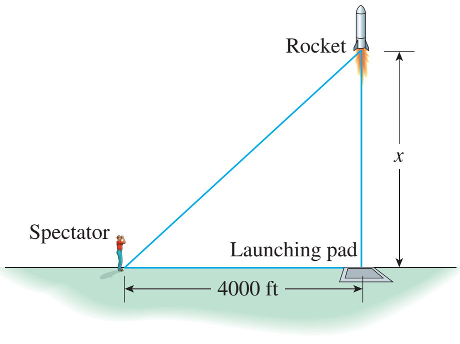
b. Using the expression obtained in part (a), find the distance between the two cruise ships 2 hr after leaving port.

43. Distance Between Two Cargo Ships Sailing north at a speed of 25 mph, Ship A leaves a port. A half hour later, Ship B leaves the same port, sailing east at a speed of 20 mph. Let t (in hours) denote the time Ship B has been at sea.

a. Find an expression in terms of t that gives the distance between the two cargo ships.

b. Use the expression obtained in part (a) to find the distance between the two cargo ships 2 hr after Ship A has left the port.

44. Watching a Rocket Launch At a distance of 4000 ft from the launch site, a spectator is observing a rocket being launched. Suppose the rocket lifts off vertically and reaches an altitude of x feet, as shown below:



a. Find an expression giving the distance between the spectator and the rocket.

b. What is the distance between the spectator and the rocket when the rocket reaches an altitude of 20,000 ft?

45. a. Show that the midpoint of the line segment joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

b. Use the result of part (a) to find the midpoint of the line segment joining the points $(-3, 2)$ and $(4, -5)$.

46. A Scavenger Hunt A tree is located 20 yd to the east and 10 yd to the north of a house. A second tree is located 10 yd to the east and 40 yd to the north of the house. The prize of a scavenger hunt is placed exactly midway between the trees.

a. Place the house at the origin of a Cartesian coordinate system, and draw a diagram depicting the situation.

b. What are the coordinates of the position of the prize?

c. How far is the prize from the house?

In Exercises 47 and 48, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

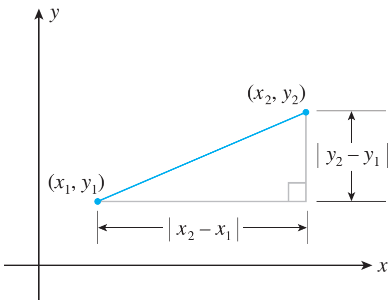
47. If the distance between the points $P_1(a, b)$ and $P_2(c, d)$ is D, then the distance between the points $P_1(a, b)$ and $P_3(kc, kd)$ $(k \neq 0)$ is given by $|k|D$.

48. The circle with equation $kx^2 + ky^2 = a^2$ lies inside the circle with equation $x^2 + y^2 = a^2$, provided that $k > 1$ and $a > 0$.

49. Let (x_1, y_1) and (x_2, y_2) be two points lying in the xy-plane. Show that the distance between the two points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Hint: Refer to the accompanying figure, and use the Pythagorean Theorem.



50. In the Cartesian coordinate system, the two axes are perpendicular to each other. Consider a coordinate system in which the x-axis and y-axis are noncollinear (that is, the axes do not lie along a straight line) and are not perpendicular to each other (see the accompanying figure).

a. Describe how a point is represented in this coordinate system by an ordered pair (x, y) of real numbers. Conversely, show how an ordered pair (x, y) of real numbers uniquely determines a point in the plane.

b. Suppose you want to find a formula for the distance between two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, in the plane. What advantage does the Cartesian coordinate system have over the coordinate system under consideration?